Holevo A. S. (Moscow, Russia) — **Quantum dynamical semigroups: nonstandard generators, stochastic representations.**

Quantum dynamical semigroups are a noncommutative analog of (sub-) Markov semigroups in classical probability: while the latter are semigroups of positive normalized maps in functional spaces, the former are semigroups of corresponding maps in operator algebras [1]. These semigroups satisfy *Markovian master equations* (M.m.e.) that are noncommutative generalization of the Kolmogorov-Chapman equations.

Let K, L_j be linear operators defined on a dense domain $\mathcal{D}$ of a Hilbert space $\mathcal{H}$, satisfying the condition

$$\sum_j \|L_j\psi\|^2 \leq 2\mathrm{Re}\,\langle\psi|K|\psi\rangle, \quad \psi \in \mathcal{D}, \tag{1}$$

in paricular, K is accretive. We assume that K is maximal accretive. Then there exists the unique minimal solution T_t, $t \geq 0$, of the Cauchy problem for the *backward* quantum M.m.e.

$$\frac{d}{dt}\langle\varphi|T_t[X]|\psi\rangle = \sum_j \langle L_j\varphi|T_t[X]|L_j\psi\rangle - \langle K\varphi|T_t[X]|\psi\rangle - \langle\varphi|T_t[X]|K\psi\rangle, \tag{2}$$

where $\varphi, \psi \in \mathcal{D}, X \in \mathfrak{L}(\mathcal{H})$, satisfying the condition $T_0[X] = X$, which is a dynamical semigroup on the algebra $\mathfrak{L}(\mathcal{H})$ of all bounded operators in $\mathcal{H}$ (cf. [2]).

If, in addition, L_j are closable and satisfy $\sum_j \left\|L_j^*\psi\right\|^2 < \infty$ for $\psi \in \mathcal{D}^*$, where $\mathcal{D}^*$ is an essential domain for K^*, then the predual semigroup $S_t^0 = (T_t)_*$ is the minimal solution of the *forward* M.m.e.

$$\frac{d}{dt}\langle\varphi|S_t^0[\omega]|\psi\rangle = \sum_j \langle L_j^*\varphi|S_t^0[\omega]|L_j^*\psi\rangle - \langle K^*\varphi|S_t^0[\omega]|\psi\rangle - \langle\varphi|S_t^0[\omega]|K\psi\rangle, \tag{3}$$

where $\varphi, \psi \in \mathcal{D}^*, \omega \in \mathfrak{T}(\mathcal{H})$, and $\mathfrak{T}(\mathcal{H}) = \mathfrak{L}(\mathcal{H})_*$ is the Banach space of trace-class operators ω in $\mathcal{H}$. There is a classical probabilistic representation

$$\langle\varphi|T_t[X]|\psi\rangle = \mathsf{M}\langle\varphi(t)|X|\psi(t)\rangle$$

via *weak-topology* solutions of the stochastic integral equation of the form

$$\psi(t) = \psi + \int_0^t \sum_j L_j\psi(s)dW_j(s) - \int_0^t K\psi(s)ds,$$

where $W_j(t); j = 1, \ldots$ are independent standard Wiener processes [2].

The semigroup is *standard* if it is a minimal solution of backward M.m.e. as above. We consider two cases of dynamical semigroups obtained by singular perturbations of the generator of a standard semigroup [3]. First, we describe an generalization of example from [4] of a standard dynamical semigroup which does not satisfy the forward M.m.e. Second, we consider an improved construction of a nonstandard dynamical semigroup.

REFERENCES

1. *Holevo A. S.* Statistical structure of quantum theory, Lect. Notes Phys. m67, Springer, Berlin 2001.
2. *Holevo A. S.* On dissipative stochastic equations in a Hilbert space, Probab. Theory Rel. Fields, 1996, vol. 104, pp. 483-500.
3. *Holevo A. S.* On singular perturbations of quantum dynamical semigroups, Math. Notes, 2018, vol. 103, № 1, pp. 133—144.
4. *Arveson W.* The domain algebra of a CP-semigroup, Pacific. J. Math., 2002, vol. 203, № 1, pp. 67-77.

This work was performed in the frame of the State Assignment of MIAN.