Smorodina N.V. (Sankt-Petersburg, Russia) — **Approximation of an evolution operator by mathematical expectations of functionals of Poisson random fields.** Consider an operator $H = -\frac{1}{2}\frac{d^2}{dx^2} + V(x)$ on the domain $W_2^2(\mathbb{R})$. Suppose that the potential V is real-valued and bounded that implies that the operator H is self-adjoint. The operator family e^{-itH} is a group of unitary operators in $L_2(\mathbb{R})$. The operator e^{-itH} maps a function $\varphi \in W_2^2(\mathbb{R})$ into the Cauchy problem solution $u(t,x)$ for the Schrödinger equation $i\,\frac{\partial u}{\partial t} = Hu$ with an initial function $u(0,x) = \varphi(x)$ (more details see in[1]). It is well known that a solution of the Cauchy problem for the heat equation $\frac{\partial u}{\partial t} = -Hu,\ u(0,x) = \varphi(x)$ admits a probabilistic representation in the form of an expectation of a Wiener process functional (Feynmann- Kac formula)

$$u(t,x) = e^{-tH}\varphi(x) = \mathbf{E}\big[\varphi(x + w(t))e^{-\int_0^t V(x+w(\tau))\,d\tau}\big], \tag{1}$$

where $w(t)$ is a standard Wiener process. Formula (1) means that that one can simulate the evolution of initial function φ under the heat semigroup e^{-tH} generating the Wiener process trajectories.

In our talk a similar approach is developed for the operator e^{-itH}. Namely, we construct a family Q_ε^t of operators in $L_2(\mathbb{R})$, depending on an additional parameter $\varepsilon > 0$ and possessing the following properties

1) for every $\varepsilon > 0$ the family Q_ε^t is a semigroup, i.e. $Q_\varepsilon^{t+s} = Q_\varepsilon^t Q_\varepsilon^s$,

2) the operator norm of the operator Q_ε^t is not greater than 1,

3) Q_ε^t is defined as expectation of a Poisson point field functional,

4) as $\varepsilon \to 0$ operators Q_ε^t approximate the operator e^{-itH} in strong operator convergence sense that is for every $\varphi \in L_2(\mathbb{R})$ we have $\|Q_\varepsilon^t\varphi - e^{-itH}\varphi\|_2 \longrightarrow 0$.

The above properties yield that that one can simulate the evolution of initial function φ under the group e^{-itH} generating the Poisson point field trajectories. It is worth to mention that. the square of wave function modulus is a density of a probability distribution. A wave function evolution generates an evolution of probability distribution density which is usually called a «quantum random walk». The suggested approach gives a theoretic possibility to simulate the «quantum random walk» by classical statistical technique. A particular case of the above construction can be found in [2].

REFERENCES

1. *Glimm J., Jaffe A.* Quantum Physics. A Functional Integral Point of View, Springer-Verlag, New York Heidelberg Berlin, 1987.
2. *Ibragimov I.A., Smorodina N.V., Faddeev M.M.* On a limit theorem related to probabilistic representation of the Cauchy problem solution for Schrödinger equation. Zap. nauchn. semin. PDMI, 2016, v.454, pp. 158-176, (in russian).

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