Tikhomirov A. N. (Syktyvkar, Russia). **Local Limit Theorems for Random Matrices.**

In this talk we review the latest results on local limit theorems for various ensembles of random matrices, including Wigner matrices, sample covariance matrices, the Ginibre – Girko matrices, and their products.

Significant progress in this area was made literally in the last decade, largely due to the work of a group of authors led by Yau H.-T. and Erdös L., see the papers [5] – [8]. The main problem is the study of the behavior of the Stieltjes transform of the empirical spectral distribution (ESD) of Hermitian random matrices of large order on the complex plane near the real axis when the distance to the real axis is inversely proportional to order n of matrix up to a logarithmic factor. Estimates of the closeness of the Stieltjes transform of the ESD of random matrix to the Stieltjes transform of the corresponding limit distribution (the semicircle law in the case of Wigner matrices, the Marchenko – Pastur distribution law in the case of sample matrices) are of order $(nv)^{-1}\log^{\beta} n$, where v is the distance to the real axis in the complex plane, β is some quantity that depends on n, but grows no faster than $\log\log n$. Such estimates make it possible to obtain an information about the local behavior of the spectrum of a random matrix, that is, about the distribution of eigenvalues in a small neighborhood of a fixed point, to find the limiting distribution for so-called spacings, i.e. distances between neighborhood eigenvalues, to estimate the closeness of eigenvalues to the corresponding quantiles of the limiting distribution (the rigidity of the spectrum) etc. In the work of the author jointly with F. Götze, A. Naumov, and D. Timushev, the main emphasis is on developing methods that allow us to obtain estimates of the order $O((nv)^{-l}\log^{\beta} n)$ under the minimal moment assumptions and with the optimal order β. Corresponding results are given in the papers [1] – [4].

REFERENCES

1. *F. Götze, A. Naumov, A. Tikhomirov, D. Timushev* A local semicircle law under weak moment conditions. (Russian) Dokl. Akad. Nauk 468 (2016), no. 1, 12–15; translation in Dokl. Math. 93 (2016), no. 3, 248–250
2. *F. Götze, A. Naumov, A. Tikhomirov, D. Timushev* On the Local Semicircular Law for Wigner Ensemble. Bernoulli, vjl. 24 N 3 (2018), 2358 – 2400.
3. *Götze F., Naumov A., Tikhomirov A.* On the Local Law for non-Hermitian Random Matrices. arXiv:1708.06950
4. *Götze, F.; Naumov, A. A.; Tikhomirov, A. N.* A local semicircle law under moment conditions: the Stieltjes transfer, rigidity and delocalization. (Russian) Teor. Veroyatn. Primen. 62 (2017), no. 1, 72–103
5. Erdös L., Yau H.-T. A dynamical approach to random matrix theory. Courant Lecture Notes in Mathematics, 28. Courant Institute of Mathematical Sciences, New York; American Mathematical Society, Providence, RI, 2017. ix+226 pp.
6. Bourgade P., Erd?os L., Yau H.-T., Yin J. Universality for a class of random band matrices. Adv. Theor. Math. Phys. 21 (2017), no. 3, 739–800.
7. Bourgade P., Yau H.-T., Yin J. Local circular law for random matrices. Probab. Theory Related Fields 159 (2014), no. 3-4, 545–595
8. *Эрдёш Л.* Универсальность вигнеровских матриц: обзор современных результатов. УМН том 66б N 3(399), стр. 67-198

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