Tikhov M.S. (Nizhni Novgorod, Russia) **Fourier method for recursive estimation of distribution function in dose-effect relationship.**

Let $\mathscr{U}^{(n)} = \{(U_1, W_1), ..., (U_n, W_n)\}$ be independent and identically distributed the pairs of random variables, where U_i is interpreted as «dose», and $W_j = I(X_j < U_j)$ is indicator of the event $(X_j < U_j)$ («effect »); the random variables $\{X_j\}_{j=1}^n$ have a common distribution function (df) $F(x) = \mathbf{P}(X_j < x)$ with the density $f(x)$, the distribution function $G(u) = \mathbf{P}(U_j < u)$ has the density $g(u) > 0$ with respect to a Lebesgue measure λ on the real line $\mathbf{R}$. It is required on the sample $\mathscr{U}^{(n)}$ to estimate the unknown distribution function $F(x)$ or its quantiles, the distribution function $G(u)$ is also unknown. This model is interpreted as a «dose-effect» relationship [1-3]. As an estimate of the distribution function $F(x)$, we take

$\hat{F}_n(x) = S_{2,n}(x)/S_{1,n}(x)$, where $S_{2,n}(x) = \sum_{j=1}^n W_j K_{b_j}(x, U_j)/n$, $S_{1,n}(x) = \sum_{j=1}^n W_j K_{b_j}(x, U_j)/n$, $K_b(x, u) = K((xu)/b)/b$, $K(x)$ is a kernel function (finite symmetric density) $\{b_j\}_{j=1}^n$ is the sequence of smoothing parameters. The technique relies on recursive representations of the estimates $\hat{F}_n(x)$ and $\hat{g}_n(x)$:

$$\hat{g}_n(x) = S_{1,n}(x) = \hat{g}_{n-1}(x) + n^{-1}[K_{b_n}(x, U_n) - \hat{g}_{n-1}(x)],$$

$$\hat{F}_n(x) = \hat{F}_{n-1}(x) + \gamma_n[W_n - \hat{F}_{n-1}(x)], \quad \gamma_n = \gamma_n(x) = (\hat{g}_n(x)\, n)^{-1} K_{b_n}(x, U_n).$$

Let $B_n = \sum_{j=1}^n b_j^{-1}$. It is shown that the conditions (see [4], p.184-185): (**D1**) $b_n \to 0$, $n^2 B_n^{-1} \to \infty$, and (**D2**) $\min\limits_{1 \le j \le n} b_j = o(B_n)$ as $n \to \infty$, and the regularity conditions on $g(x)$ and $f(x)$ that can be efficiently checked in many cases in order to ensure that the sequence $nB_n^{-1/2}(\hat{F}_n(x_0) - \mathbf{E}(\hat{F}_n(x_0)))$ are asymptotically normal $N(0, F(x_0)(1 - F(x_0))\|K\|^2/g(x_0))$ as $n \to \infty$.

We also consider the estimation problem of the distribution function $F(x)$ in the convolution model using the Fourier method, and in addition to the estimation distribution function using the theory of reproduced kernel.

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