Zhitlukhin M. V. (Steklov Mathematical institute, Moscow, Russia), **Borovkov K. A.**
On the maximum of discretely sampled fractional Brownian motion with small Hurst parameter.

Let $\{B_t^H\}_{t\geq 0}$ denote the fractional Brownian motion with Hurst parameter $H \in (0,1]$, which is a continuous Gaussian process with $B_0^H = 0$, zero mean and covariance function $\mathrm{E}(B_s^H B_t^H) = \frac{1}{2}(s^{2H} + t^{2H} - |t-s|^{2H})$, $s, t \geq 0$.

We show that the distribution of the maximum $\overline{B^{H,\mathbb{T}}} = \max_{t\in\mathbb{T}} B_t^H$ of the fractional Brownian motion with Hurst parameter $H \to 0$ over an n-point set $\mathbb{T} \subset [0,1]$ can be approximated by the normal law with mean $\sqrt{\ln n}$ and variance $1/2$ provided that $n \to \infty$ slowly enough and the points in $\mathbb{T}$ are not too close to each other.

The main results as follows. Let $H_k \in (0,1]$ be such that $H_k \to 0$ as $k \to \infty$, and $\mathbb{T}_k = \{t_{k,i}\}_{i=1}^{n_k}$ be a sequence of finite subsets of $(0,1]$, $t_{k,1} < \cdots < t_{k,n_k}$, such that $n_k \to \infty$, $\delta_k := \min_{1\leq i\leq n_k}(t_i - t_{i-1}) \to 0$, where $t_0 = 0$.

Denote by $\preccurlyeq$ the stochastic order relation for random variables, i.e. $\xi \preccurlyeq \eta$ iff $\mathrm{P}(\xi \leq x) \geq \mathrm{P}(\eta \leq x)$, for any $x \in \mathbb{R}$. Let $o_P(1)$ stand for a sequence of random variables converging to zero in probability.

Theorem. (i) *If* $H_k(\ln n_k)^{1/2} \to 0$ *and* $H_k \ln(n_k\delta_k) \to 0$ *as* $k \to \infty$ *then*

$$\overline{B^{H_k,\mathbb{T}_k}} \preccurlyeq \sqrt{\ln n_k} + \zeta_0/\sqrt{2} + o_P(1).$$

(ii) *If* $H_k(\ln n_k)^2 \to 0$ *and* $H_k \ln \delta_k \to 0$ *as* $k \to \infty$, *then*

$$\overline{B^{H_k,\mathbb{T}_k}} \succcurlyeq \sqrt{\ln n_k} + \zeta_0/\sqrt{2} + o_P(1).$$

Thus, under the assumptions from part (ii), one has

$$\overline{B^{H_k,\mathbb{T}_k}} - \sqrt{\ln n_k} \xrightarrow{\mathrm{d}} Z/\sqrt{2} \quad \text{as} \quad k \to \infty,$$

where Z has the standard normal distribution. The conditions $H_k \ln(n_k\delta_k) \to 0$, $H_k \ln \delta_k \to 0$ are automatically met in the case of uniform grids $\mathbb{T}_k$.