Afanasyev V.I. (Moscow, Russia). **Functional limit theorems for decomposable branching processes with two particle types.**

Consider a branching Galton-Watson process with particles of two types. Suppose that a particle of the first type generates descendants of both types, and in the same quantities, and a particle of the second type generates descendants of only its type.

Let $\varphi(\cdot)$ and $\psi(\cdot)$ be generating functions of nonnegative integer random variables ξ and η. Suppose that the maximum step of distribution of the random variable ξ is 1. Also, suppose that $\mathbf{E}\xi = 1$, $\mathbf{Var}\xi :=\sigma_1^2 \in (0,\infty)$ and $\mathbf{E}\eta = 1$, $\mathbf{Var}\eta :=2b_2 \in (0,\infty)$.

We introduce generating functions for the progeny of a particle of the first or second types, respectively, of the branching process under consideration: for $s_1, s_2 \geq 0$

$$f_1(s_1,s_2) = \varphi(s_1 s_2), \qquad f_2(s_1,s_2) = \psi(s_2);$$

denote ξ_n and η_n the number of particles of the first and second types, respectively, in the n-th generation of the branching process under consideration. It is supposed that $\xi_0 = 1$ and $\eta_0 = 0$. Set $\Sigma_2 = \sum_{n=1}^{\infty} \eta_n$.

Let $\left\{l_0^+(t),\ t \geq 0\right\}$ be a local time of Brownian excursion and $\{Y(t),\ t \geq 0\}$ be a Feller diffusion. Set $S = \int_0^\infty Y(b_2 t)\,dt$ and introduce the probability densities for $y > 0$

$$p_1(y) = \frac{\mathbf{P}^{(1)}\left(\sqrt[4]{S} > y\right)}{\mathbf{E}^{(1)}\sqrt[4]{S}}, \qquad p_2(y) = \frac{2}{\mathbf{E}^{(1)}\sqrt[4]{S}} \frac{\mathbf{P}^{(1)}\left(\sqrt[4]{S} > y^{-1/2}\right)}{y^{3/2}}$$

(the upper index of $\mathbf{P}$ and $\mathbf{E}$ means that $Y(0) = 1$).

Theorem 1. *As* $N \to \infty$

$$\left\{ \frac{\xi_{\lfloor t\sqrt[4]{N} \rfloor}}{\sqrt[4]{N}},\ t \geq 0 \ \middle|\ \Sigma_2 > N \right\} \xrightarrow{D} \left\{ \frac{\sigma_1}{2\nu} l_0^+\left(\frac{\sigma_1}{2} t\nu\right),\ t \geq 0 \right\},$$

where ν *is a random variable with probability density* p_1 *and independent of the process* $\left\{l_0^+(t),\ t \geq 0\right\}$, *the symbol* $\xrightarrow{D}$ *means convergence in distribution in the space* $D[0,\infty)$.

Theorem 2. *As* $N \to \infty$

$$\left\{ \frac{\eta_{\lfloor t\sqrt{N} \rfloor}}{\sqrt{N}},\ t > 0 \ \middle|\ \Sigma_2 > N \right\} \xrightarrow{D} \left\{ Y(b_2 t),\ t > 0 \mid S > 1 \right\},$$

here the random variable $Y(0)$ *has a probability density* p_1 *and the symbol* $\xrightarrow{D}$ *means convergence in distribution in the space* $D(0,\infty)$.

REFERENCES

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