**Belopolskaya Ya.I.** (Sankt-Petersburg, Russia). **A probabilistic interpretation of the MHD-Burgers system as a system of nonlinear forward Kolmogorov equations.**

We derive closed systems of stochastic equations associated with systems of parabolic equations with cross-diffusion and interpret them as systems of forward Kolmogorov equations [1]-[3]. We derive formulas of the Feynman- Kac type to construct probabilistic representations of mild solutions of the forward Cauchy problem such systems. We develop a general theory and demonstrate its results constructing a probabilistic interpretation of the Cauchy problem solution to one of the simplest magneto-hydrodynamics systems, namely, the MHD-Burgers system

$$\frac{\partial v}{\partial t} + v \cdot \nabla v = \frac{\sigma^2}{2}\Delta v + (\nabla \times B) \times B, \quad v(0,y) = v_0(y), \tag{1}$$

$$\frac{\partial B}{\partial t} = \frac{\mu^2}{2}\Delta B + \nabla \times (v \times B), \quad B(0,y) = B_0(y). \tag{2}$$

Here $v$ is the fluid velocity and $B$ is the magnetic field. Let $w_m(t), m = 1, \dots, 6$, be independent Wiener processes, $\xi_{0m}$ be random variables with $P(\xi_{0m} \in dy) = u_{0m}(dy)$, independent of $w_m(t)$, $u_{0m} = v_{0m}, m = 1, 2, 3$, $u_{0m} = B_{0m}, m = 4, 5, 6$.

We define stochastic processes $\xi_m(t), \eta_m(t)$ by relations

$$\xi^m(t) = \xi_{0m} + \sigma w_m(t), m = 1, 2, 3, \quad \xi_m(t) = \xi_{0m} + \mu w_m(t), m = 4, 5, 6, \tag{3}$$

$$\eta_m(t) = 1 + \int_0^t c_m(u(\theta, \xi_m(\theta)), \nabla u(\theta, \xi_m(\theta)))\eta_m(\theta) d\theta, \tag{4}$$

$u = (u_1, \dots, u_6)$, $u_m = v_m, m = 1, 2, 3, \quad u_m = B_m, m = 4, 5, 6$,

$$\int_{R^d} h_m(y) u_m(t, dy) = E\left[h_m(\xi_m(t))\eta_m(t)\right], \quad m = 1, \dots, 6, \tag{5}$$

where $c_m : R^6 \times (R^6 \otimes R^3) \to R$ are defined from (1), (2). By the Riesz theorem we deduce from (5)

$$u_m(t, y) = \int_{R^d} p_m(0, x, t, y) u_{0m}(x) dx + \int_0^t \int_{R^d} \tilde{c}^u_m(t, z) p_m(\theta, z, t, y) u_m(\theta, z) dz d\theta, \tag{6}$$

where $p_m(0, x, t, y)$ is the density of the process $\xi_m(t)$ transition probability, $\tilde{c}^u_m(t, z) = c_m(u(t, z), \nabla u(t, z))$ and $u_m(t, dy) = u_m(t, y)dy$.

**Theorem.** *Let $u_{0m} \in W^{1,1}(R^3)$. Then there exists a unique solution of the stochastic system (3),(4), (6) defined on an interval $[0, T]$. Functions $u_m \in L^1([0, T], W^{1,1}(R^d)) \cap L^1([0, T], L^\infty(R^d))$ given by (6) are the unique mild solution of the Cauchy problem (1),(2).*

In the one dimensional case a similar result was obtained in [1].

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