Bordag L.A. (Zittau, Germany) — **Optimization problem for a portfolio with an illiquid asset in the case of an exponential utility function.**

We study an optimization problem for a portfolio with an illiquid, a risky and a risk-free asset in the framework of continuous time. Problems of such type lead to a three dimensional nonlinear Hamilton-Jacobi-Bellman (HJB) equation on the value function. The corresponding solution of the three dimensional partial differential equation (PDE) describes the value function and the investment - consumption strategies. In this framework we suppose that the illiquid asset is sold in an exogenous random moment of time T with a prescribed liquidation time distribution. Earlier we studied similar optimization problems with a HARA (hyperbolic absolute risk aversion) utility function and with a logarithmic utility function in [2-5].

In this paper we study the optimization problem with two types of exponential utility functions, with negative and positive ones (denoted as EXPn and EXPp), which are equivalent from an economical point of view. Exponential utility functions belong to the type of CARA (constant absolute risk aversion) utility functions. It is well known that both the logarithmical (LOG) and the negative exponential (EXPn) utility functions are connected with the HARA utility function by means of a limiting procedure: in the first case the parameter of the HARA utility function is going to zero and in the second case to infinity. In our previous papers [3-4], devoted to the optimization problem with a general HARA and LOG utility functions we proved that also the corresponding analytical and Lie algebraic structures are connected with the same limiting procedure. In this paper we show that the case of EXPn utility function differs from the case of the HARA utility function and has its own special Lie algebraic structure which is not connected to the HARA case by the limiting procedure. We carry out the Lie group analysis of the three dimensional PDEs for the cases positive and negative exponential utility functions and we are able to obtain the admitted symmetry algebras. A short and comprehensive introduction in this method as well as applications to other PDEs arising in financial mathematics is given in [1].

Here we prove also explicitly that both optimization problems with negative and positive exponential utility functions are connected by a one-to-one analytical substitution and are identical from the economical, analytical or Lie algebraic point of view. We use admitted Lie algebras to prove equivalence of the both problems as well as to obtain reductions of the studied PDEs equations. We provide the complete set of nonequivalent group invariant reductions of the three dimensional PDE corresponding to the optimization problem with the EXPn utility function to two dimensional PDEs in accordance with an optimal system of sub algebras of the admitted Lie algebra. We prove that in one case the invariant reduction is consistent with the boundary condition. The two dimensional PDE is more convenient for applications of numerical methods as the original three dimensional PDE. Because of the uniqueness of the solution of the HJB equation we can use the reduced two dimensional PDE to study the properties of the optimal solution and the investment - consumption strategies.

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