Danekyants A. G., Neumerzhitskaia N. V. (Rostov-on-Don, Russia) **On rational and irrational interpolating martingale measures.**

This report is a continuation of [1,2] and is related to the question of the existence of non-degenerate weakly interpolating martingale measures (n.d.w.i.m.m.) of a one-step market with a discounted stock price $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$ (the definition of n.d.w.i.m.m. can be found in [1,2]). The process Z is defined on the countable outcome space $\Omega = \{\omega_1, \omega_2, \dots\}$; $\mathcal{F}_0 = \{\Omega, \emptyset\}$, $\mathcal{F}_1$ is the set of all subsets of Ω; $a := Z_0$, $b_i := Z_1(\omega_i)$, $i = 1, 2, \dots$, $\mathbf{b} := (b_1, b_2, \dots)$. It is assumed that the market in question is arbitrage-free, that is, it admits martingale measures $P = (p_1, p_2, \dots)$ on $(\Omega, \mathcal{F}_1)$, for which $p_i = P(\omega_i) > 0$, $i = 1, 2, \dots$, and the process $Z = (Z_k, \mathcal{F}_k, P)_{k=0}^1$ is a martingale.

A non-zero sequence $\mathbf{r} = (r_1, r_2, \dots)$ will be called finite if its components are rational and among them only a finite number are non-zero. For a sequence of real numbers $\mathbf{d} = (d_1, d_2, \dots)$ we denote by $\mathcal{L}(\mathbf{d})$ a set of numbers of the form $\sum r_i d_i$, where $\mathbf{r}$ runs through all finite sequences.

Theorem. 1. Suppose that the number a is irrational, and the sequence $\mathbf{b}$ contains an infinite number of different rational terms. If $a \notin \mathcal{L}(\mathbf{b})$, then the market under consideration admits n.d.w.i.m.m.

2. Let the number a and all the terms of the sequence $\mathbf{b}$ be rational. If a martingale measure $P = (p_1, p_2, \dots)$ is such that $\mathcal{L}(\mathbf{P})$ consists only of irrational numbers, then P is n.d.w.i.m.m.

The first item of the theorem is a generalization of the result from [2]. Part 2 of the theorem is supported by the following example.

Example. Let $(d_1, d_2, \dots)$ be an arbitrary positive sequence such that $\mathcal{L}(\mathbf{d})$ consists only of irrational numbers (for example, if a number t is transcendental, then the sequence $(t, t^2, t^3 \dots))$ satisfies this property). Find the sequence $(c_1, c_2, \dots)$ of positive rational numbers such that $\sum c_i d_i = 1$. Put $p_i = c_i d_i$. Let a be an arbitrary rational number. Find a sequence of rational numbers $\mathbf{b}$ such that $\sum b_i p_i = a$. Then $P = (p_1, p_2, \dots)$ is n.d.w.i.m.m.

REFERENCES

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The research is supported by RFBR (Grant 19-01-00451).