Dimitrov D.V. (Moscow, Russia) — **Statistical estimation of the Kullback-Leibler divergence.**

Consider i.i.d. random vectors $X_1, X_2, \ldots$, and i.i.d. random vectors $Y_1, Y_2, \ldots$, with $law(X_1) = law(X)$ and $law(Y_1) = law(Y)$ for some random vectors X and Y taking values in $\mathbb{R}^d$, $d \geq 1$ and having distributions P_X and P_Y, respectively. Assume that X and Y have densities $p = \frac{d\mathsf{P}_X}{d\mu}$ and $q = \frac{d\mathsf{P}_Y}{d\mu}$ with respect to Lebesgue measure μ. Suppose also that $\{X_i, Y_i, i \in \mathbb{N}\}$ are independent. We are interested in statistical estimation of the Kullback - Leibler divergence $D(\mathsf{P}_X||\mathsf{P}_Y) = \int_{\mathbb{R}^d} p(x) \log\left(\frac{p(x)}{q(x)}\right) \mu(dx)$ constructed by means of observations $\mathbb{X}_n := \{X_1, \ldots, X_n\}$ and $\mathbb{Y}_m := \{Y_1, \ldots, Y_m\}$, $n, m \in \mathbb{N}$. Proposed estimates involve the specified nearest neighbor statistics, analogues of which were introduced in the pioneering paper [2]. The authors of [3] indicated the occurrence of gaps in the known proofs concerning the limit behavior of such statistics. This issue has attracted our attention and motivated our study of the declared asymptotic properties. Wide conditions are provided to guarantee asymptotic unbiasedness and L^2-consistency of such estimates. In particular, the established results are valid for estimates of the Kullback - Leibler divergence between any two Gaussian measures in $\mathbb{R}^d$ with nondegenerate covariance matrices. As a byproduct we obtain new results concerning the Kozachenko-Leonenko estimators of the Shannon differential entropy (see, e.g., [1]).

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