Gliklikh Yu.E. (Voronezh, Russia) **Stochastic equations with current velocities and osmotic velocities (mean derivatives).**

All preliminaries about mean derivatives, in particular, about current velocities (symmetric mean derivatives D_S), osmotic velocities (antisymmetric mean derivatives D_A) and quadratic mean derivatives D_2 can be found in [1].

Let the diffusion coefficient, i.e., the field of symmetric matrices $(a^{ij}(x))$, be smooth, autonomous and positive definite. Since all matrices $(\alpha^{ij}(x))$ are non-degenerate and smooth, there exist the smooth field of converse symmetric and positive definite matrices (α_{ij}). Hence this field can be used as a new Riemannian metric $\alpha(\cdot,\cdot) = \alpha_{ij}dx^i \otimes dx^j$ on $\mathbb{R}^n$. The volume form of metric $\alpha(\,\cdot\,,\cdot\,)$ is $\Lambda_\alpha = \sqrt{\det(\alpha_{ij}(x))}\, dx^1 \wedge \cdots \wedge dx^n$.

Denote by symbol $\rho(t,x)$ the density of probablistic distribution of random element $\xi(t)$ with respect to the volume form $dt \wedge \Lambda_\alpha$ on $\mathbb{R} \times \mathbb{R}^n$, i.e., for every continuous bounded function $f\colon [0,T] \times \mathbb{R}^n \to \mathbb{R}$ the relation $\int_0^T E(f(t,\xi(t)))\,dt = \int_0^T \left(\int_\Omega f(t,\xi(t))d\mathbb{P}\right) dt = \int_{\mathbb{R}\times\mathbb{R}^n} f(t,x)\rho(t,x)dt \wedge \Lambda_\alpha$ holds.

Let a Borel vector field $v(t,x)$ and a Borel field of symmetric positive semi-definite matrices $\alpha(t,x)$ be given on $\mathbb{R}^n$. The system of the form

$$\begin{cases} D_S\xi(t) = v(t,\xi(t)), \\ D_2\xi(t) = \alpha(t,\xi(t)), \end{cases} \tag{1}$$

where the equalities are fulfilled a.s., is called the first order equation with current velocities.

Everywhere below we suppose that the fields v and α are smooth and all matrices $\alpha(x)$ are autonomous and positive definite. If, in addition, v, α and partial derivatives of the coefficients of (a^{ij}) satisfy the Ito inequality and the density ρ of initial value is smooth and nowhere equal to zero, it is proved that (1) has a solution (see [2]).

Lemma 1 *Let $\rho(t,x)$, $v(t,x)$, $\alpha(x)$ and Λ_α be the same as above and the solution of (1) exists. Then the flow of vector field $(1, v(t,x))$ on $\mathbb{R}\times\mathbb{R}^n$ preserves the volume form $\rho(t,x)dt\wedge \Lambda_\alpha$, i.e., the Lie derivative $L_{(1,v(t,x))}\rho(t,x)dt \wedge \Lambda_\alpha = 0$.*

The system

$$\begin{cases} D_A\xi(t) = u(t,\xi(t)) \\ D_2\xi(t) = \alpha(\xi(t)) \end{cases}, \tag{2}$$

where $u(t,x)$ is a Borel vector field and α is as above, is called the first order differential equation with osmotic velocity.

By the use of the properties of osmotic velocity, properties of quadratic velocity and the Stokes theorem we can find $\rho(t,x)$ that can be the density of solution of (2). Introduce $p(t,x) = \log\rho(t,x)$.

Assumption 1. *We suppose that for every $t \in [0,T]$ the integral $\int_{\mathbb{R}\times\mathbb{R}^n} e^{p(t,x)}dt \wedge \Lambda_\alpha$ is finite, i.e., is equal to a certain finite constant $C_{(t)}$ that is C^∞-smooth in t.*

Theorem 1 *If Assumption 1 is satisfied, under the above-mentioned hypotheses equation* (2) *has a solution for a certain initial value having the density smooth and nowhere equal to zero and depending on the right-hand side. This solution is not unique.*

The proof uses Lemma 1 in order to find the current velocity of solution.

1. *Gliklikh Yu.E.* Global and stochastic analysis with applications to mathematical physics.- London: Springer-Verlag.- 2011.- 460 pp.
2. *Azarina S.V., Gliklikh Yu.E.* On the Solvability of Nonautonomous Stochastic Differential Equations with Current Velocities Mathematical Notes, 2016, vol. 100, no. 1, pp. 3 – 10.

This work was supported by the RFBR (project 18-01-00048).