Kudryavtsev O.E. (Rostov-on-Don, Russia) — **On approaches to pricing European and American lookback options.**

A lookback option is an exotic derivative whose payoff depends on the extrema of a certain underlying asset price reached within the option lifetime. Lookback options pricing under various processes has been dealt with by a host of researchers. The most efficient approaches are deterministic methods that use integral transforms techniques (see e.g. [1,3]). We will focus on a class of Lévy models which are popular among practitioners in finance. On the one side, Lévy processes include the famous Black-Scholes model, on the other, they may admit jumps.

From the practical viewpoint, the trader needs to price lookback options, not at the initial time only, but during the whole time period up to the expiration date. Let $S_t = e^{X_t}$ be the price of an underlying asset driven by an exponential Lévy model. Then the time-T_1 price of a European lookback with expiration date T_2 conditional on $X_{T_1} = x$ and the extrema of the underlying asset price observed prior to the current time T_1 can be considered as a seasoned lookback with the time remaining to expiration $T = T_2 - T_1$ and prefixed maximum or minimum. Recall that prices of seasoned European floating and fixed strike lookback put with prefixed maximum $U = e^y$ and minimum $D = e^z$, respectively, can be expressed as follows.

$$V_{\text{fl}}(T,x,y) = E^x\big[e^{-rT}(e^{\max\{\bar{X}_T,y\}} - e^{X_T})\big], \quad V_{\text{fx}}(T,x,z) = E^x\big[e^{-rT}(K - e^{\min\{\underline{X}_T,z\}})_+\big],$$

where K is a strike price, $\bar{X}_t$ and $\underline{X}_t$ are supremum and infimum processes. Both type of options could be computed by the direct implementation of the generalized Monte Carlo method based on the Wiener-Hopf factorization developed in [4]. Numerical experiments show that the Monte Carlo for lookback options under Lévy models is sufficiently fast and accurate in comparison to the deterministic methods from [1,3].

The second part of the talk deals with finite difference methods for pricing American lookbacks in the Black-Scholes framework. Unlike European lookback options, American lookback options cannot be priced by closed-form formulae, even in the Black-Scholes model, and require the use of numerical methods.

The price of an American floating strike put on a dividend paying stock can be expressed in terms of the solution U to the following variational inequality (see [2]):

$$\begin{cases} \max\left(e^y - e^x, \frac{\partial U}{\partial t} + \frac{\sigma^2}{2}\frac{\partial^2 U}{\partial x^2} + (r - q - \frac{\sigma^2}{2})\frac{\partial U}{\partial x} - rU\right) = 0 \quad t < T, x < y, \\ \frac{\partial U}{\partial y}(t,x,x) = 0, \\ U(T,x,y) = e^y - e^x, x \le y, \end{cases}$$

where r is the interest rate, and q is a continuous dividend rate. The main idea behind new approach is to represent reduce one dimension of the problem using the following representation of the price $U(t,x,y) = e^y - e^x + e^y F(t, x-y)$, at $x \le y$, where $F(t,x) \ge 0$ as $x \le 0$. We apply the Wiener-Hopf method to prove that the function F is non-decreasing in x on the negative half-axis.

REFERENCES

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