Kuznetsov D. F. (Saint-Petersburg, Russia) — **Strong approximation of iterated Ito and Stratonovich stochastic integrals.**

This work continues the research started in [1] on the development of efficient methods of mean square approximation of iterated Ito and Stratonovich stochastic integrals which can be applied to the numerical solution of Ito SDEs.

Theorem. *Let* $\psi_1(\tau),\ldots,\psi_k(\tau)$ *be continuous functions at* $[t,T]$, $\phi_j(\tau)$ *is a complete orthonormalized polynomial or trigonometric basis in* $L_2([t,T])$, *and* $i_1,\ldots,i_k=0,1,\ldots,m$. *Then* $I_{T,t}^k = \text{l.i.m.}_{p_1,\ldots,p_k\to\infty} I_{T,t}^{p_1\ldots p_k}$ $(k\in\mathbf{N})$, $J_{T,t}^k=\text{l.i.m.}_{p\to\infty} J_{T,t}^{k,p}$ $(k\le 5)$, *moreover* $\mathbf{E}(I_{T,t}-I_{T,t}^{p_1\ldots p_k})^2\le k!(\|K\|^2-\sum_{j_1}^{p_1}\ldots\sum_{j_k=0}^{p_k} C_{j_k\ldots j_1}^2)$ $\forall T-t\in(0,1)$, *where*

$$I_{T,t}^k=\int_t^T\psi_k(t_k)\ldots\int_t^{t_2}\psi_1(t_1)d\mathbf{W}_{t_1}^{(i_1)}\ldots d\mathbf{W}_{t_k}^{(i_k)},\ J_{T,t}^k=\int_t^T\ldots\int_t^{t_2}\circ d\mathbf{W}_{t_1}^{(i_1)}\ldots\circ d\mathbf{W}_{t_k}^{(i_k)},$$

$$I_{T,t}^{p_1\ldots p_k}=\sum_{j_1=0}^{p_1}\ldots\sum_{j_k=0}^{p_k}C_{j_k\ldots j_1}\left(\prod_{\ell=1}^k\zeta_{j_\ell}^{(i_\ell)}-\underset{N\to\infty}{\text{l.i.m.}}\sum_{(\ell_1,\ldots,\ell_k)\in G_k}\prod_{s=1}^k\phi_{j_s}(\tau_{\ell_s})\Delta\mathbf{W}_{\tau_{\ell_s}}^{(i_s)}\right),$$

$$J_{T,t}^{k,p}=\sum_{j_1,\ldots,j_k=0}^{p}C_{j_k\ldots j_1}\prod_{\ell=1}^k\zeta_{j_\ell}^{(i_\ell)},\ C_{j_k\ldots j_1}=\int_{[t,T]^k}K(t_1,\ldots,t_k)\prod_{\ell=1}^k\phi_{j_\ell}(t_\ell)dt_1\ldots dt_k,$$

$\|\cdot\|$ *is a norm in* $L_2([t,T]^k)$, d *and* $\circ d$ *are Ito and Stratonovich differentials respectively,* $K(t_1,\ldots,t_k)=I\{t_1<\ldots<t_k\}\psi_1(t_1)\ldots\psi_k(t_k)$, $\mathbf{W}_\tau^{(i)}$ $(i=1,\ldots,m)$ *are independent standard Wiener processes,* $\mathbf{W}_\tau^{(0)}=\tau$, $\Delta\mathbf{W}_{\tau_j}^{(i)}=\mathbf{W}_{\tau_{j+1}}^{(i)}-\mathbf{W}_{\tau_j}^{(i)}$, $\zeta_j^{(i)}=\int_t^T\phi_j(\tau)d\mathbf{W}_\tau^{(i)}$ $(i\ne 0)$ are *i.i.d.* $\text{N}(0,1)$*-random variables,* $t=\tau_0<\ldots<\tau_N=T$, $\max_{0\le j\le N-1}(\tau_{j+1}-\tau_j)\to 0$ *as* $N\to\infty$, $H_k=\{(\ell_1,\ldots,\ell_k):\ell_1,\ldots,\ell_k=0,1,\ldots,N-1\}$, $L_k=\{(\ell_1,\ldots,\ell_k):\ell_1,\ldots,\ell_k=0,1,\ldots,N-1;\ \ell_g\ne\ell_r(g\ne r);g,r=1,\ldots,k\}$, $G_k=H_k\backslash L_k$.

REFERENCES

1. *Kuznetsov D.F.* Stochastic Differential Equations: Theory and Practice of Numerical Solution. Electr. J. Differ. Equations Control Proc., 2018, № 4, pp. A.1–A.1073. http://diffjournal.spbu.ru/EN/numbers/2018.4/article.2.1.html