Lykov A.A., Malyshev V.A., Melikian M.V. (Moscow, Russia) — **Stability problems for infinite linear chains.**

We consider infinite number of point particles $... < x_k < x_{k+1} < ..., k \in Z$, on R (infinite chain of oscillators) with formal Hamiltonian

$$H = \sum_k \frac{v_k^2}{2} + \frac{\omega_0^2}{2} \sum_k (x_k - ka)^2 + \frac{\omega_1^2}{2} \sum_k (x_{k+1} - x_k - a)^2, a > 0,$$

$$y = \{y_k(t) = x_k(t) - ka\},\ v(t) = \{\dot{y}_k = \dot{x}_k\}, \quad M(t) = \sup_{k \in \mathbb{Z}} |y_k(t)|.$$

and present some results concerning stability (in l_∞) of its fixed point (zero energy point) with respect to various perturbations.

Theorem 1. Suppose that $y(0)$, $v(0) \in l_2(\mathbb{Z})$, then:

1. If $\omega_0 > 0$, then $\sup_{t \geqslant 0} M(t) < \infty$.
2. If $\omega_0 = 0$ then for all $t \geqslant 0$ the following inequality holds:

$$M(t) \leqslant \frac{2}{\sqrt{\omega_1}} ||v(0)||_2 \sqrt{t} + ||y(0)||_2$$

   but for all $\delta > 1/2$ there is initial condition $y(0) = 0$, $v(0) \in l_2(\mathbb{Z})$ such that (Γ is the gamma function) $\lim_{t \to \infty} \frac{y_0(t)}{\sqrt{t}} \ln^\delta t = \Gamma(\delta) > 0$.

Theorem 2. If $\omega_0 = 0$ and $v(0) = 0$ then:

1. If $y(0) \in l_\infty(\mathbb{Z})$, then for all $t \geqslant 0$: $M(t) \leqslant \left(c\sqrt{t} + 2\right) M(0)$, for some constant $c \geqslant 0$.
2. If $y_k(0)$, $k \in \mathbb{Z}$ is i.i.d and bounded in k with probability one (i.e. $\sup_k |y_k(0)| < \infty$ a.s.) then for all $n \in \mathbb{Z}$ $\mathrm{P}(\limsup_{t \to \infty} y_n(t) = +\infty) = \mathrm{P}(\liminf_{t \to \infty} y_n(t) = -\infty) = 1$.

REFERENCES

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