Martynov G. V. (IITP RAS, Moscow, Russia) — **New Cramer-von Mises multivariate uniformity test for large dimensions.**

Traditional form of the Cramér-von Mises goodness-of fit test of distribution uniformity on $[0,1]^m$ is

$$\omega_n^2 = n \int_{[0,1]^m} \left(F_n(t_1, ..., t_m) - \prod_{j=1}^{m} t_j \right)^2 dt_1 ... dt_m.$$

Here $F_n(t_1, ..., t_m) = \frac{1}{n} \sum_{i=1}^{n} \prod_{j=1}^{m} I_{T_{i,j} < t_j}$ is the empirical distribution function, $T_i = (T_{i,j},\ j = 1, ...m,)\ i = 1, ..., n$, are n observations of a random m-vector T. Particularly, the statistic ω_n^2 is considered in [2]. However, the use of this statistic is difficult even for small values of m. Moreover, its distribution degenerates with $m \to \infty$. To eliminate this drawback, it is proposed to use the generalized uniform distribution function $\tilde{F}(\mathbf{t}) = t_1^{\alpha_1} \cdot ... \cdot t_m^{\alpha_m},\ \alpha_1 \geq -1, ..., \alpha_m \geq -1$. The corresponding empirical distribution function will be $\tilde{F}_n(t_1, ..., t_m) = \frac{1}{n} \sum_{i=1}^{n} \prod_{j=1}^{m} I_{T_{i,j} < t_j^{\alpha_j}}$. Then the statistic ω_n^2 is transformed to the form

$$\tilde{\omega}_n^2 = n \int_{[0,1]^m} \left(\tilde{F}_n(t_1, ..., t_m) - \prod_{j=1}^{m} t_j^{\alpha_j} \right)^2 dt_1 ... dt_m.$$

To stabilize the distribution $\tilde{\omega}_n^2$, the constants $\alpha_m > -1$ should tend sufficiently quickly toward zero when dimension m tends to ∞. The report discusses how to calculate the limiting distributions of the statistics $\tilde{\omega}_n^2$ at finite values of m. Initial ideas were given in [3]. The limiting distribution $\tilde{\omega}_n^2$ at $m = \infty$ was investigated at [4] using the results from [1]. Although the limiting distributions of statistics at finite values m are calculated by exact methods, but the statistics values themselves, both ω_n^2 and $\tilde{\omega}_n^2$, can be calculated only by the Monte Carlo method. The effectiveness of using this method in the case under consideration is shown.

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