Nasyrov F.S. (Ufa, Russia) — **On strong solutions of stochastic differential equations.**

Given a filtered probability space $(\Omega, F, (F_t)_{t\in[t_0,T]}, P)$ satisfying the usual condition on which a standart Wiener process $W(s)$ is defined, by Q denote σ-algebra of predictable sets. Let $B(D)$, $D \subseteq R^n$, be a Borel σ-algebra.
The aim of this lecture is to prove the following statements.
1. Consider the ordinary differential equation $y' = f(t, W(t), y+W(t))$, $t \in [t_0, T]$, $y(t_0) = y_0$, where $f(t, v, y) = f(t, v, y, \omega)$ is $Q \times B(R^2)$-measurable function. Suppose that

$$|f(t, v, y)| \leq n(t), \text{ where } n(t) = n(t, \omega) \text{ is integrable in } t. \tag{1}$$

Then ordinary differential equation (1) admits a strong solution.
2. Suppose that $X(s)$ is an arbitrary continuous function. We make the following assumptions. (a) The equation $(\varphi^*)'_v = \sigma(t, v, \varphi^*)$ has a general solution $\phi^*(t, v, C(t))$. (b) The measurable function $f(t, y) = \frac{B(t,X(t),\varphi^*(t,X(t),y))-(\varphi^*)'_t(t,X(t),y)}{\sigma(t,X(t),\varphi^*(t,X(t),y))}$ satisfyes inequality (1) with the nonrandom integrable function $n(t)$. Under (a) and (b) the determined equation

$$\xi(t) - \xi(t_0) = \int_{t_0}^{t} \sigma(s, X(s), \xi(s)) * dX(s) + \int_{t_0}^{t} B(s, X(s), \xi(s))\, ds. \tag{2}$$

with respect to symmetrical integral has a solution.
3. Let assumptions (a) and (b) hold for the Borel function $\sigma(t, u, \varphi)$ and $Q \times B(R^2)$ - measurable function $B(t, u, \varphi, \omega)$. Then Stratonovich stochastic differential equation (2) with $X(s)$, replacing to the Wiener process $W(s)$, has a strong solution.
4. Consider an Ito equation $d\xi(t) = \sigma(s, W(s), \xi(s))dW(s) + B(s, W(s), \xi(s))ds$, $\xi(0) = \xi_0$. Let assumptions (a) and (b) of section **3** hold. If the continuous function $\sigma(t, u, \varphi)$ has continuous partial derivatives $\sigma'_u(t, u, \varphi)$ and $\sigma'_\varphi(t, u, \varphi)$, then the strong solution $\xi(t)= \phi^*(t, W(t), C(t))$ exists and a random function $\phi(t, W(t))\equiv\phi^*(t, W(t), C(t))$ with $P = 1$ satisfyes the equation $\phi'_t(t, W(t)) = -\frac{1}{2}\phi''_{uu}(t, W(t)) + b(t, W(t), \phi(t, W(t)))$, $t \in [t_0, T]$.

REFERENCES

1. *Nasyrov F.S.* Local times, symmetrical integrals and stochastic analyses, Moscow, Fizmatlit, 2011, – 211 p. (in russian)