Rodionov I.V. (V.A. Trapeznikov Institute of Control Sciences RAS, Moscow, Russia)
— **Parameter estimation of distribution tails.**

In this talk we focus on the problem of parameter estimation of distribution tails. The problem of tail estimation is central to statistics of extremes of independent observations. The generally accepted approach to estimate the distribution tail in this theory is semi-parametric and based on Pickands-Balkema-de Haan theorem (cf. [1], [2]) reducing this problem to the problem of estimating the extreme value index (see for details [3]). The mentioned approach works well for distributions with power-law tails, that often appears in finance and insurance. However, one cannot distinguish between the distribution tails with exponential rate of decrease using this approach, [4]. Moreover, the conditions of Pickands-Balkema-de Haan theorem are not satisfied for the large class of distributions, in particular, distributions with logarithmic tails. Therefore, it becomes necessary to propose the general method of tail estimation not based on the latter theorem such that it can be applied to most distributions used in practice.

Let $\mathbf{X} = (X_1, \ldots, X_n)$ be independent identically distributed random variables with a continuous distribution function F. Let $X_{(1)} \leq \ldots \leq X_{(n)}$ be the nth order statistics corresponding to $\mathbf{X}$. Evidently, only the largest order statistics can be used in the problem of tail estimation. Assume that F belongs to the parametric class of continuous distribution tails $\mathcal{F} = \{F_\theta, \theta \in \Theta\}$, $\Theta \subset \mathbb{R}$. How to select the appropriate parametric class, see [4], [5]. In order to propose the method of estimating the parameter θ introduce the following statistic

$$R_{k,n}(\theta) = \ln(1 - F_\theta(X_{(n-k)})) - \frac{1}{k} \sum_{i=n-k+1}^{n} \ln(1 - F_\theta(X_{(i)})).$$

Then the estimator of the parameter θ

$$\widehat{\theta}_{k,n} = \arg\{\theta : \ R_{k,n}(\theta) = 1\}$$

is consistent as $k \to \infty, k/n \to 0, n \to \infty$ under some weak conditions imposed on $\mathcal{F}$, [6].

In this talk we discuss the properties of the proposed method, i.e. uniqueness of a solution of the equation $R_{k,n}(\theta) = 1$, consistency and asymptotic normality of $\widehat{\theta}_{k,n}$, its modification for Weibull-tail and log-Weibull-tail index estimation, among the others.

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