**Rytova A. I.** (Moscow, Russia) — **Branching walk with an infinite number of initial particles and heavy tails.**

The evolution of a random field $n(t,\cdot)$ of particles on $\mathbb{Z}^d$, $d \geq 1$, where $n(t,y)$ is the number of particles at the moment $t$ at point $y$, is considered. The evolution of the field includes the particles random walk on the lattice and branching at the origin. At the initial moment of time, at each point $x$ there is a single particle, which determines the subpopulation $n_x(t,\cdot)$ of particles that originate from it, i.e. $n(0,\cdot) \equiv 1$ and $n(t,\cdot) \equiv \sum_{x\in\mathbb{Z}^d} n_x(t,\cdot)$. Such a model was considered in [1] for the BRW with a finite variance of jumps, where duality was established for the first moments with the BRW model with a single initial particle. In this paper, the random walk is assumed to be heavy-tailed, the variance of the jumps becomes infinite, the remaining assumptions are preserved. The purpose of the study is the asymptotics, as $t \to \infty$, of the average numbers of particles of a population $\mathsf{E}\, n(t,y)$ and of subpopulations $\mathsf{E}\, n_x(t,y)$ at the point $y \in \mathbb{Z}^d$ depending on the parameters of walking and branching, the various combinations of which allow classifying the BRW as subcritical, critical or supercritical (see [2]).

Due to works [1-4], the following theorem was obtained

**Theorem 1** *For BRW on $\mathbb{Z}^d$, $d \geq 1$, with the parameter $\alpha \in (0,2)$ that determines the heaviness of the random walk tails, as $t \to \infty$, it is valid*

$$\mathsf{E}\, n(t,y) \sim C(y)\, v(t), \quad \mathsf{E}\, n_x(t,y) \sim C(x,y)\, u(t),$$

| *BRW criticality* | *combination of $d$ and $\alpha$* | $v(t)$ | $u(t)$ |
|---|---|---|---|
| *supercritical* | | $e^{\lambda t}$ | $e^{\lambda t}$ |
| *critical* | *(a), (b)* | $1$ | $t^{-1/\alpha}$ |
| | *(c)* | $t^{d/\alpha-1}$ | $t^{d/\alpha-2}$ |
| | *(d)* | $t \ln^{-1} t$ | $\ln^{-1} t$ |
| | *(e)* | $t$ | $1$ |
| *subcritical* | *(a)* | $t^{1/\alpha-1}$ | $t^{1/\alpha-2}$ |
| | *(b)* | $\ln^{-1} t$ | $t^{-1}\ln^{-2} t$ |
| | *(c), (d), (e)* | $1$ | $t^{-d/\alpha}$ |

*where $\lambda$, $C(x,y)$, $C(y)$, are some positive constants, and cases are identified:* $(a)\, d = 1, \alpha \in (1,2)$, $(b)\, d = 1, \alpha = 1$, $(c)\, d = 1, \alpha \in (1/2,1)$, *or* $d = 2, \alpha \in (1,2)$, *or* $d = 3, \alpha \in (3/2,2)$, $(d)\, d = 1, \alpha = 1/2$, *or* $d = 2, \alpha = 1$, *or* $d = 3, \alpha = 3/2$, $(e)\, d = 1, \alpha \in (0,1/2)$, *or* $d = 2, \alpha \in (0,1)$, *or* $d = 3, \alpha \in (0,3/2)$, *or* $d \geq 4, \alpha \in (0,2)$.

## REFERENCES


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