Shiryaeva L. K. (Samara state university of economics, Russia). **On rotated versions of the three-parameter Grubbs copula.**

Consider Grubbs statistics $T_{n,(1)} = (\overline{X} - \min\{X_i\})/S$ и $T_n^{(1)} = (\max\{X_i\} - \overline{X})/S$, as calculated for a normal sample of size n (see [1]). Assume that in the sample $\{X_i\}_{i=1}^n$ there is one abnormal observation (outlier) and its number is unknown. We believe that the outlier differs from the other observations by the shift α and the scale parameters $\nu > 0$. We denote

$$G_{n,(1)}(t;\alpha,\nu) = P(T_{n,(1)} < t),\ G_n^{(1)}(t;\alpha,\nu) = P(T_n^{(1)} < t),$$

$$\Upsilon_n(x,y;\alpha,\nu) = P(T_{n,(1)} < x, T_n^{(1)} < y).$$

Recursive relationships for the marginal distribution functions $G_{n,(1)}(\cdot)$, $G_n^{(1)}(\cdot)$ and the joint distribution function $\Upsilon_{n)}(\cdot)$ were found in [2]. According to Sclar's theorem [3] the copula C^{Gr}, extracted from a joint distribution Υ_n, is

$$C^{Gr}(G_{n,(1)}(x;\alpha,\nu), G_n^{(1)}(y;\alpha,\nu); n,\alpha,\nu) = \Upsilon_n(x,y;\alpha,\nu).$$

Grubbs copula describes negative interdependencies between random variables. Rotated on 90° и 270° versions of the copula can be used to model positive interdependencies, i.e. $C_{90}^{Gr}(u,v;n,\alpha,\nu) = v - C^{Gr}(1-u,v;n,\alpha,\nu)$ and $C_{270}^{Gr}(u,v;n,\alpha,\nu) = u - C^{Gr}(u,1-v;n,\alpha,\nu)$. The following theorem describes the properties of the rotated versions of the copula.

Theorem 1. *Let* $\Xi_n^{(90)} = \{0 \le u \le 1; \delta_n(1-u;\alpha,\nu) \le v \le 1\}$, $\Xi_n^{(270)} = \{0 \le u \le 1; 0 \le v \le 1 - \delta_n(u;\alpha,\nu)\}$ *and* $M(u,v) = \min(u,v)$ *is maximum copula. Then for* $n \ge 3$

$$C_{90}^{Gr}(u,v;n,\alpha,\nu) = M(u,v),\ \forall(u,v) \in \Xi_n^{(90)};$$
$$C_{270}^{Gr}(u,v;n) = M(u,v),\ \forall(u,v) \in \Xi_n^{(270)},$$

where $\delta_n(u;\alpha,\nu) = G_n^{(1)}(\theta_n(\phi_{n,(1)}(u,v;n,\alpha,\nu)))$ *and* $\phi_{n,(1)}(\cdot)$ *is quasi-inverce of* $G_{n,(1)}(\cdot)$.

Corollary. *In the case* $n = 3$ *for* $\forall\alpha$ *and* $\nu > 0$ *it is valid*

$$C_{90}^{Gr}(u,v;3,\alpha,\nu) = C_{270}^{Gr}(u,v;3,\alpha,\nu) = M(u,v),\ \ \forall(u,v) \in [0,1]^2.$$

The work is also up to

Lemma. *Let statistics* $T_{3,(1)}$ *and* $T_3^{(1)}$ *are calculated from a set of continuous random variables* X_1, X_2, X_3 *with an arbitrary distribution. Then the copula, described the joined distribution of* $T_{3,(1)}$ *and* $T_3^{(1)}$, *is minimum copula, but rotated on* 90° *and* 270° *its versions coincide with maximum copula.*

REFERENCES

1. *Grubbs F.E.* Sample criteria for testing outlying observations. Ann.Math. Statist.,1950, V.21, N 1, p. 27–58.
2. *Shiryaeva L. K.* On distrubution of Grubbs' statistics in case of normal sample with outlier. Russian Math. (Iz. VUZ), 2017, V.61, N.4, p. 72–88.
3. *Nelsen R.B.* An Introduction to Copulas, Springer, New York, 2006.