Smorodina N.V. (St.Petersburg, Russia) — **A construction of reflecting Lévy processes.**

We consider one-dimensional special type Markov processes that are Lévy processes taking value on an interval (for definiteness — on the interval $[0, \pi]$) and reflecting from boundary points. Informally we can describe the process behavior in the following way. Given a process we equip it with some additional reflection mechanism. Namely, the process starts from the point $x \in [0, \pi]$ and moves in accordance with its trajectory, until it reaches the boundary of the interval. At this time the process reflects from the boundary, leaves on it "the jump of momentum" and continues to move on. The jumps of momentum are accumulated at each point of the boundary with time. It is obvious that we need a precise definition of the reflection because the Lévy processes trajectories are non-differentiable or or moreover they may have pure jump-type trajectories. A standard way to define the reflection for a continuous trajectory is connected with a solution of the so-called Skorokhod problem. Since the Lévy process trajectories in general are not continuous we use another approach based on ideas of the generalized function theory. Let us describe the main ideas of this approach. Let $\xi(t),\ \xi(0) = 0$ be a Lévy process, and $\xi_x(t) = x + \xi(t)$. We consider a test function f and the operation of reflection is transferred to it. That means that we consider some new "reflected" function $\widetilde{f}$. Then we define a reflected trajectory by the formula $\mathbf{E} f(\widetilde{\xi_x}(t)) = \mathbf{E}\widetilde{f}(\xi_x(t))$. Such a reflected process $\widetilde{\xi_x}(t)$ is a Markov process so the last formula automatically defines all finite-dimensional distributions.

We consider the family $\widetilde{\xi_x}(t),\ x \in [0, \pi]$ of the reflected precesses in the above sense. The evolution of a reflected process is described by two operator families $R^t,\ Q^t$. R^t is an operator semigroup acting in $L_2[0, \pi]$ which describes the evolution of probability law in $[0, \pi]$. The elements of the second family we also consider as operators. Q^t maps a function defined on the boundary (that is two-point set $\{0, \pi\}$) into a function from $L_2[0, \pi]$. The family Q^t describes the evolution of the mean value of accumulated momentum at boundary points.

For the Lévy processes we define not only average accumulated momentum but also random accumulated momentum $\mathcal{Q}^t(\xi(\cdot))$ that is accumulated momentum of the "individual" trajectory so that for every function $g = \{g(0), g(\pi)\}$ we have $(Q^t g)(x) = \mathbf{E}[\mathcal{Q}^t(\xi(\cdot))g](x)$. The random accumulated momentum is always a nonnegative additive functional of the process trajectory.

The random accumulated momentum of the reflected Wiener process increases only at the moments when the process reaches the boundary and can be expressed through the local time of the reflected Wiener process. For an arbitrary Lévy process it is not the case even if the local time exists.

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