Tikhov M. S., Grishin V. A. (NNSU, Nizhny Novgorod, Russia). **A simple bias reduction method in nonparametric distribution function estimation.**

This work continues the research considered in [1], [2] on estimating the distribution function in a dose-response relationship. Namely, let $\mathcal{U}^{(n)} = \{(U_i, W_i)\}_{i=1}^n$ be a sample of pair (U, W), where $W_i = I(X_i < U_i)$ is an indicator of the event $(X_i < U_i)$ and the sequences $(U_i)_{1 \le i \le n}$ and $(X_i)_{1 \le i \le n}$ are independent. The pair (X, U) has a joint distribution function $F(x)G(u)$ and a joint density $f(x)g(u)$. We are interested in the estimation of the distribution function $F(x)$ of the random variable X. As an estimate of $F(x)$, statistics $\hat{F}_n(x) = S_{2n}(x)/S_{1n}(x)$, $S_{2n}(x) = n^{-1} \sum\limits_{i=1}^{n} W_i K_h(x - U_i)$, $S_{2n}(x) = n^{-1} \sum\limits_{i=1}^{n} W_i K_h(x - U_i)$, $S_{1n}(x) = n^{-1} \sum\limits_{i=1}^{n} K_h(x - U_i)$, are usually applied, $\mathbf{W} = (W_1, ..., W_n)^T$, $K_h(x) = h^{-1}K(h^{-1}x)$, $h = h(n)$ is a smoothing parameter called the bandwidth [1]. We define the diagonal matrix $\mathbf{A} = \mathrm{diag}\{K_h(x - U_1), ..., K_h(x - U_n)\}$, and the octahedral norm $||\,\boldsymbol{x}\,|| = \sum\limits_{j=1}^{n} |\,x_j|$ for a vector $\boldsymbol{x} = (x_1, ..., x_n)^T$, and define a norm of the operator $||\,\mathbf{A}\,|| = \sup\limits_{||\,\boldsymbol{x}\,||\,=1} ||\,\mathbf{A}\boldsymbol{x}\,||$. Set $\mathbf{A}_k = \mathbf{I} - (\mathbf{I} - \mathbf{A})^k$, $\hat{S}_{2n}^{(k)}(x) = \mathbf{A}_k \mathbf{W}$, $||\,\mathbf{I} - \mathbf{A}\,|| = \lambda$. As $n \to \infty$, we have $0 < \lambda < 1$, i.e. the operator $\mathbf{I} - \mathbf{A}$ is contracting, thus as $k \to \infty$ the estimator $\hat{S}_{2n}^{(\infty)}(x)$ to the function $F(x)g(x)$ at $n \to \infty$ will be unbiased, and $\mathbf{D}(\hat{S}_{2n}^{(k)}(x)) \sim \dfrac{F(x)g(x)\,||\,K\,||_2^2}{nh}$. Similarly, the bias of the estimator $S_{1n}(x)$ for the density $g(x)$ is decreased alongside with the estimator $\hat{F}_n(x)$. The report also considers the method of stochastic approximation (SA) working out an estimate of the distribution function $F(x)$. The asymptotic normality of the constructed SA-estimate is proved.

REFERENCES

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