Dyakonova E.E., Vatutin V.A. (Moscow, Russia) — **Evolution of a weakly subcritical branching process in random environment: population size at the initial stage.**

Let $\mathcal{Z} := \{Z_n, n = 0, 1, ...,\}$ be a branching process in random environment (BPRE) specified by a sequence of random independent and identically distributed generation functions $\{f_n(s), n = 1, 2, ...\}$ (see [1] for a detailed treatment of properties of such processes). Let

$$X_n = \log f_n'(1), \qquad \eta_n = \frac{f_n''(1)}{(f_n'(1))^2}.$$

The BPRE is called *weakly subcritical* if $\mathbb{E}[X_1] < 0$ and there is a number $0 < \beta < 1$ such that $\mathbb{E}[X_1 e^{\beta X_1}] = 0$.

Theorem 0.1 *Let $\mathcal{Z}$ be a weakly subcritical BPRE satisfying the conditions $\mathbb{E}[X_1^2 e^{\beta X_1}] < \infty$ and $\mathbf{E}[(\log^+ \eta_1)^{2+\varepsilon}] < \infty$ for some $\varepsilon > 0$. If $r = r(n) \to \infty$ in such a way that $r = o(n)$ then, as $n \to \infty$*

$$\mathcal{L}\left(\frac{1}{\sqrt{r}} \log Z_r \middle|\, Z_n > 0\right) \to \mathcal{L}(B_1),$$

where $\{B_t, t \geq 0\}$ is a Brownian motion conditioned to stay nonnegative for all $t \geq 0$ (see [2] for the respective definition).

This result complements Theorem 1 in [3] dealing with the case $r \sim tn$ for $t \in (0, 1)$. Observe that an analog of this statement for the critical BPRE was established in [4].

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