

## ABSTRACT OF THE TALK

### On the Ruin Problem with Investment when the Risky Asset is a Semimartingale

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The estimation of the probability of ruin of insurance companies is a fundamental problem for market actors. Classically, a compound Poisson process with drift was used to model the value of an insurance company and in that case, under some assumptions, the probability of ruin decreases at least as an exponential function of the initial capital.

Over time, the compound Poisson process has been replaced by more complex models. In a first generalisation, the value of the company is modeled by a Lévy process and then the ruin probability behaves essentially like the tail of the Lévy measure and, in the light-tailed case, this means that this probability decreases again at least as an exponential function of the initial capital.

To generalise even further, it was assumed that insurance companies invest their capital in a financial market. The main question is then: how does the probability of ruin changes with this additional source of risk?

In this setting, the value of an insurance company with initial capital  $y > 0$ , denoted by  $Y = (Y_t)_{t \geq 0}$ , is given as the solution of the following linear stochastic differential equation

$$Y_t = y + X_t + \int_0^t Y_s dR_s, \text{ for all } t \geq 0, \quad (1)$$

where  $X = (X_t)_{t \geq 0}$  and  $R = (R_t)_{t \geq 0}$  are two independent Lévy processes. Here the process  $X$  represents the profit and loss of the business activity and  $R$  represents the return of the investment and  $y$  is initial capital of the insurance company.

The main problem then concerns the study of the stopping time defined by

$$\tau(y) = \inf\{t \geq 0 \mid Y_t < 0\}$$

with  $\inf\{\emptyset\} = +\infty$  and the evaluation of the ruin probability  $P(\tau(y) \leq T)$  before time  $T > 0$ , and the ultimate ruin probability  $P(\tau(y) < +\infty)$ .

The ruin problem with investment for Lévy processes was studied by Asmussen (2000), Paulsen (1993, 2002, 2008), Kalashnikov and Norberg (2002), Frolova and Kabanov (2002), Pergamentschikov and Zeitouni (2006), Kabanov, Pergamentschikov (2016, 2018). There where obtained upper and lower bounds for the ultimate ruin probability and it was proved that, under reasonable conditions, this probability decreases as  $y^{-\beta}$  with some  $\beta > 0$ .

In this talk, we study the ruin problem with investment in a general framework where the business part  $X$  is a Lévy process and the return on investment  $R$  is a semimartingale. Under some conditions, we obtain upper and lower bounds on the finite and infinite time ruin probabilities as well as the logarithmic asymptotic for them. When  $R$  is a Lévy process, we retrieve some well-known results. The conditions and the bounds involve the moments of the exponential integral functionals of the process  $R$  which were studied in the case of additive processes in Vostrikova, Salminen (2018,2019).

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