Zhitlukhin M.V. (Moscow, Russia). **Asymptotically optimal strategies in a market model with competition**

We consider a multistep game model where several players (investors) compete for distribution of payments made by several assets. The goal of the paper is to study strategies which are optimal on the infinite time horizon in the sense that they do not lose to any other strategies of the competitors. Problems of this type were considered for the first time in the paper [1] for a particular model with discrete time; then they were studied (also in discrete time), for example, in the papers [2, 3]. The present papers considers a more general model, both in discrete and continuous time.

For simplicity, in this abstract, the model is described only for the case of discrete time. Suppose that on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t=1}^{\infty}, \mathrm{P})$ are given N adapted non-negative sequences A_t^n, which represent payments of the assets n at time t. By a strategy of player m (where $m = 1, \dots, M$) we call a sequence of $\mathcal{F}_{t-1} \otimes \mathcal{B}(\mathbb{R}_+^M)$-measurable functions $l_t^{m,n}(\omega, y)\colon \Omega \times \mathbb{R}_+^M \to \mathbb{R}_+$, which expresses the amount of wealth invested by this player into asset n at time t; the argument $y \in \mathbb{R}_+^M$ corresponds to the vector of wealth of all the players at time $t-1$. It is assumed that the players select the values $l_t^{m,n}$ simultaneously and independently of each other.

Let the vector $\bar{Y}_0 = (Y_0^1, \dots, Y_0^M)$ with $Y_0^m > 0$ denote a given initial amount of the wealth of the players at the initial moment of time. Then, by definition, the amounts of wealth at the subsequent moments of time are specified by the relation

$$Y_t^m = Y_{t-1}^m - \sum_n l_t^{m,n}(\bar{Y}_t) + \sum_n \left(\frac{l_t^{m,n}(\bar{Y}_t)}{\sum_k l_t^{k,n}(\bar{Y}_t)} A_t^n \right). \tag{1}$$

The first sum in the right-hand side is equal to the investment expenses of player m, the second sum is the profit he receives from the assets (the payment from each asset is split between the players proportionally to their investments in it).

Let $r_t^m = Y_t^m / \sum_k Y_t^k$ be the fraction of wealth of player m in the total wealth. The goal of the paper is to find a strategy such that if player m uses it, then $\inf_t r_t^m > 0$ a.s. for any strategies of the other players, i.e. the fraction of his wealth remains bounded away from zero all the time.

In the paper, such a strategy is found in an explicit form in a general model with continuous time. Moreover, it is shown that it is essentially unique: all the strategies which have this property are asymptotically close to it in some sense.

REFERENCES

1. R. Amir, I. V. Evstigneev, K. R. Schenk-Hoppé. Asset market games of survival: a synthesis of evolutionary and dynamic games. *Annals of Finance*, 9(2):121–144, 2013.

2. L. Blume, D. Easley. Evolution and market behavior. *Journal of Economic Theory*, 58(1):9–40, 1992.

3. I. Evstigneev, T. Hens, K. R. Schenk-Hoppé. Evolutionary behavioral finance. In E. Haven et al., *The Handbook of Post Crisis Financial Modelling*, 214–234. Palgrave Macmillan UK, 2016.

4. M. V. Zhitlukhin. Asymptotic distribution of capital in a model of an investment market with competition. *arXiv:1811.12491*.